Markscheme

November 2019

Mathematics

On-screen examination

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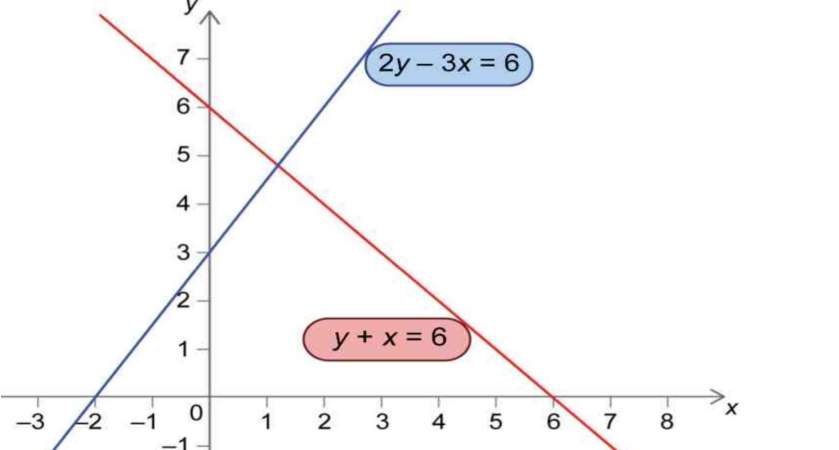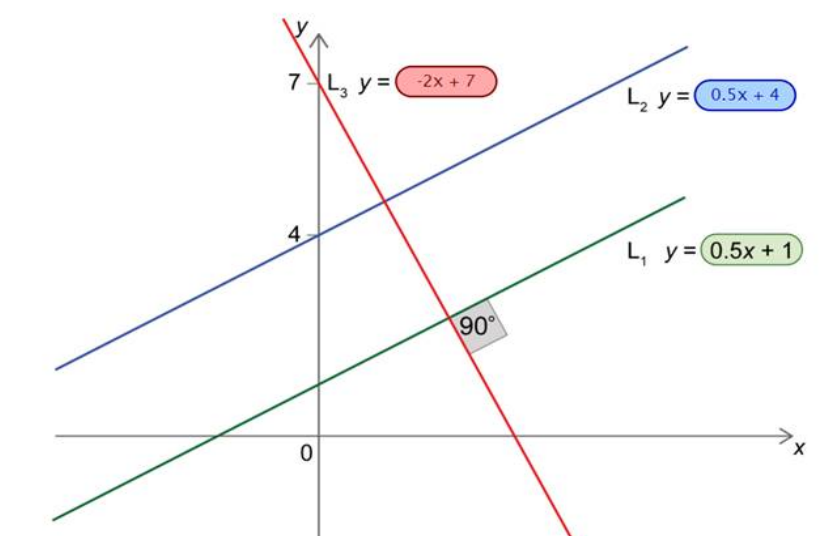
Error Carried Forward (ECF) marks
Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.
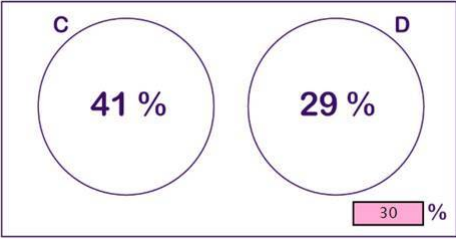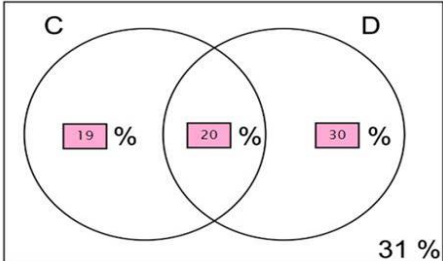   a) **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.
   b) If an answer resulting from **ECF** is inappropriate (*eg*, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.
   c) If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.
   d) To award **ECF** marks for a question part, **there must be working present for that part**.
   e) **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.
   f) A misread (**MR**) is an error**. ECF** is normally awarded.

General points

   a) As this is an international examination, accept all alternative forms of **notation,** for example 1.9 and 1,9 or 1 000 or 1.000. However **DO NOT ACCEPT** incorrect
       mathematical notation e.g x^2 for x^2 unless noted otherwise in the MS.
   b) Accept notation errors in intermediate steps.
   c) Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer,**
       then the last mark cannot be awarded.
   d) In the case when a correct result is obtained using incorrect seen method, do not award the mark for the result.
   e) Where candidates have written two solutions to a question, mark the first solution.
   f) In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** (or

       equivalent) e.g. $\dfrac{1}{2}$ **OR** 1/2 **OR** $1 \div 2$ and $\dfrac{x}{2}$ $x\,/\,2$ **OR** $x \div 2$

   g) In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.
   h) Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.
   i) Accept seeing equation not in-line.
   j) Calculator screenshots are accepted as working steps. And when a calculator screenshot is taken, accept not seeing the whole operation.
   k) In task 2 and 3 where the markscheme is set out in a table then, unless noted otherwise, awarding the highest mark in a category includes all the lower marks in
       that category. It is probably best to look for the top category mark answer and if you don't find it look at the next mark down.
   l) ACCEPT using the correct values regardless their previous result

Question		Answers	Notes	Total
1	a	.1 correctly place $y + x = 6$ .2 correctly place $2y - 3x = 6$		**2**
	b	.1 correctly write equation of L2 as $(y =)0.5x + 4$ OE .2 correctly write equation of L3 as $(y =) -2x + 7$ OE		**2**

Question		Answers	Notes	Total
2	a	**AM1 (adding volumes)**	**AM1 (adding volumes)**	
		.1 correct Volume for one part	.1 50x20x30 (=30000) OR 35x20x30(=21000)	
		.2 correct Volume for second part	.2 20x15x30 (=9000) OR 40x15x30(=18000)	
		.3 correctly add their volumes	.3 (their30000+their9000) OR (their21000+their18000) =39000 (ft³)	
		AM2 (Subtracting volumes)	**AM2 (Subtracting volumes)**	
		.1 correct volume including empty part	.1 50x40x30 (=60000)	
		.2 correct volume of empty part	.2 35x20x30 (=21000)	**3**
		.3 correctly subtract their volumes	.3 (their60000-their21000) =39000 (ft³)	
		AM3 (Area x depth)	**AM3 (Area x depth)**	
		.1 correct Area for one part	.1 50x20 (=1000) OR 20x15 (=300)	
		.2 correct Area for the face	.2 (50x20 + 20x15) = 1300	
		.3 correctly multiply the total of their areas by 30	.3 (their1300 x30) =39000 (ft³)	
	b	.1 add employees	.1 105+70+90+75+60 (=400)	
		.2 divide by 5	.2 their400 / 5	**3**
		.3 correct result after dividing the sum by 5	.3 80	
	c	.1 correctly substitute their39000 and their80	.1 $(P=)\dfrac{(6\times their39000+500\times their80)}{9000}$	**2**
		.2 correctly calculate their P after their substitution involving at least one multiplication in numerator	.2 their 30(.444…)	

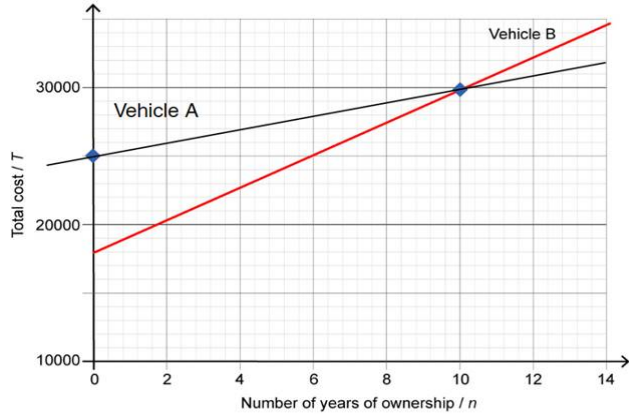| Question		Answers	Notes	Total
3	a	correctly place 30 %		1
	b	correctly state a reason for mutually exclusive related to sets representation	No intersection ACCEPT they are separate or apart WTTE $A \cap B = 0$ *or* $\varnothing$ DO NOT ACCEPT No girl selected both	1
	c	sum of percentages add up to more than 100%	ACCEPT 20% selected both	1
	d	.1 correct working step .2 correct value of intersection .3 correct value of cat only .4 correct value of dog only	.1 39 + 50 - 69 or 39+50+31-100 or 50-20=30 and 39-20=19 .1 ACCEPT seeing this step in 3c .2 20 .3 19 .4 30 	4
	e	.1 0.69 seen .2 probability of adopt from both branches seen .3 correct result after adding their multiplied values	.1 ACCEPT 69 seen on tree diagram .2 0.6 x 0.7(=0.42) OE and 0.4 x 0.69(=0.276, ACCEPT 0.28) OE .3 their 0.696 OE, ACCEPT 0.7 OE only if .2 seen	3

Question		Answers	Notes	Total
4	a	multiply 140 by 5 and divide by 7 AG 100	$\frac{140}{7} \times 5$, ACCEPT $\frac{5}{7} = \frac{100}{140}$ OE ACCEPT $5 \times 20 : 7 \times 20 (= 100 : 140)$ ACCEPT $100 / 20 = 5$ *and* $140 / 20 = 7$	1
	b	.1 correctly substitute 1 and 0.5 .2 correct answer	.1 140 x 1 + 100 x 0.5 .2 190	2
	c	.1 correct working step .2 correct result after adding the three ratios multiplied by value .3 correctly divide 1760 by 11 OR multiply 1760 by 9 .4 multiply ratios by their 160 OR divide their15840 by 11 .5 correctly calculate their number of coins	.1 2n x 0.5 + 4n x 1 + 3n x 2 or 2 x 0.5 + 4 x 1 + 3 x 2 .1 ACCEPT 9 seen .2 11n or 11 .3 $(\frac{1760}{11} =)160$ or $(9 \times 1760 =)15840$ .4 2xtheir160 and 4xtheir160 and 3xtheir160 or 320,640,480 or 9xtheir160 OR 15840/11. ACCEPT their160 being 1760/9 .5 their1440 DO NOT ACCEPT .4 or .5 if their160 is not the same number	5

Question		Answers	Notes	Total
5	**a**	(0,12)	ACCEPT 0,12 x=0, y=12 0;12 DO NOT ACCEPT 12 C=12	**1**
	b	.1 correctly set equation to solve .2 correctly factorize their equation or substitute into quadratic formula .3 correct coordinates of their point A .4 correct coordinates of their point B	.1 $x^2 - 8x + 12 = 0$ .2 their $(x-2)(x-6)$ .3 (A=) their (2,0) .3 DO NOT ACCEPT if negative .4 (B=) their (6,0) .4 DO NOT ACCEPT if negative	**4**
	c	correctly write coordinates of their point B after reflection on the y-axis	ACCEPT their-6,0 x=their-6 , y=0 DO NOT ACCEPT -6 Their(-6,0) if positive	**1**

Question		Answers	Notes	Total
6	**a**	180 − 140	ACCEPT 360 − 90 − 90 − 140 140 and 40 are complementary or add up to 180 Starting with XOY=40 and reach XSY=140	**1**
	b	.1 correctly substitute 40 and 6372 into arc formula .2 correctly calculate their answer after substitution into arc length formula	.1 $\dfrac{40}{360}2\pi(6372)$ OE .2 their 1416π or $4448.495\ldots$	**2**
	c	**AM1 (using only trig ratios)** .1 select the angle to be used for trig ratio .2 correctly substitute 20 into the trig ratio .3 correctly rearrange their trig ratio for ON .4 correctly calculate their result after rearranging trig ratio .5 correctly round their answer to the nearest km **AM2 (using Pythagoras)** .1 correctly substitute XN and r correctly into trig ratio .2 correctly rearrange for XN .3 correctly substitute their r and theirXN into Pythagoras or use tan70 .4 correctly calculate their result after using Pythagoras .5 correctly round their answer to the nearest km	**AM1 (using only trig ratios)** .1 20 or 70 seen .2 $\cos 20 = \dfrac{ON}{6372}$ or $\sin 70 = \dfrac{ON}{6372}$ .3 (ON=)6372cos20 or 6372sin70 .4 Their 5 987.7(21…) .4 DO NOT ACCEPT using sin70 or cos20=0.9 .4 Award as ecf only if their ON is less than 6372 .5 Their 5 988 **AM2 (using Pythagoras)** .1 $\dfrac{XN}{6372} = \sin 20\ or\ \cos 70$ .2 $(XN =)6372\sin 20\ or\ 6372\cos 70\ or\ [2166,2179.4]$ .3 $(ON =)\sqrt{6372^2 - their\,2179.35...^2}$ .3 ACCEPT their 2179.35 not in $[2166,2179.4]$ only if there is working .4 Their 5 987.7(21…) .4 DO NOT ACCEPT using sin20 or cos70=0.3 .4 Award as ecf only if their ON is less than 6372 .5 Their 5 988	**5**
	d	correctly subtract theirON from 6372 answer	(6372 – their 5988 =) 384 ACCEPT answers in the range [382, 384.3] Award as ecf only if positive	**1**

	e	.1 correctly substitute theirMN and 6372 into the surface area of spherical cap formula	.1 $2\pi \times 6372 \times$ their 384 or their 4893696π ACCEPT using $\pi = \dfrac{22}{7}$ or 3.14	**3**
		.2 correctly calculate their result after substitution into the surface area of spherical cap formula .3 correctly write their answer rounded to 2 sf and in standard form	.2 their 15 384 154(.37…) .2 ACCEPT answers in the range [15 286 173,15 386 011] .3 their 1.5×10^7	
	f	.1 correctly substitute 6372 into the Surface Area of sphere formula .2 divide their 7e by their surface area of sphere .3 correctly calculate their ratio as percentage	.1 $4\pi \times 6372^2$ or 510 224 605(.2..) .2 $\dfrac{\text{their } 1.5 \times 10^7}{\text{their } 510\ 224\ 605}$ OE $e.g.$ $\dfrac{\text{their } 1.5}{\text{their } 51}$ or their 0.029 .3 their 2.9(…) ACCEPT 3(%)	**3**

Question		Answers	Notes	Total
7	a	**AM1** .1 recognise 80 .2 multiply 80 % by 31250 .3 correctly calculate their result after multiplying 31250 by their percentage **AM2** .1 multiply 20 % by 31250 .2 subtract 31250×0.2 from 31250 .3 correctly calculate their result after subtraction of their reduction from 31250	**AM1** .1 80 or 0.8 seen .2 0.8×31250 OE .3 their25000 **AM2** .1 31250×0.2 or 6250 seen .2 $31250 - 31250 \times 0.2$ or $31250 - 6250$ .3 their25000	**3**
	b	.1 multiply 14000 by 0.035 .2 correct answer	.1 14000×0.035 seen .2 490	**2**
	c	.1 multiply 1190 by 7 or 8330 seen .2 add 18 000 AG £26 330		**2**
	d	$T = 490n + 25000$ .1 their25 000 as the y intercept .2 their490 as the gradient	ACCEPT $y = 490x + 25000$, $y = (14000 \times 0.035)x + 25000$	**2**

7	e			**(1 mark)**	**(2 marks)**		
			Factors (F)	**Two** factors from: Vehicle cost; ACCEPT Total cost Fuel (cost or usage of per mile) The number of miles driven per year Number of years of ownership or distances travelled maintenance cost or long term cost Effect on the environment	**Three** factors from: Vehicle cost; ACCEPT Total cost Fuel (cost or usage of per mile) The number of miles driven per year Number of years of ownership or distances travelled maintenance cost or long term cost Effect on the environment		
			Graph (G)	Attempt to plot their V_A line from earlier results Ex: correct slope or T-intercept from their earlier results or At least two points plotted satisfy their earlier results with acceptable accuracy	Correctly plot their V_A line from their earlier results ACCEPT T-intercept $\in their]24000,26000[$ and n at intersection $\in their]9.5,10.5[$		**10**
			Number of years (N)	Attempt to determine after how many years their V_A and V_B will have the same total cost Ex: Attempt to solve simultaneously or trial and improvement OR Correctly calculate the total cost after a specific number of years for both vehicles	Correctly determine after how many years their V_A and V_B will have the same total cost Ex: Correctly calculate n=their10 after solving simultaneously or trial and improvement OR n=their10 and their graph intersects at n=their10		
			Justify (J)	Weak justification Ex: V_A is better for the environment even if its initial cost is more OR V_A is better for the environment AND it will cost less on the long run or overall cost will be less OR	Good justification Ex: V_A is better for the environment AND V_A is better if years of ownership are more than their n at point of intersection or V_B is better if years of ownership are less than their n point of intersection on the graph		

			V_A is better for the environment AND a justification matching their graph OR V_B is better as their calculations show V_B costs less even if not good for environment	
		Comment on Accuracy (A)	Not very accurate with weak reason. Ex: I rounded the number of years to get the total cost Didn't include other factors of owning a vehicle e.g. running costs OR Accurate with good reason. Ex: The values did not require rounding so the total cost is accurate. The values were whole numbers OR Sensible rounding used	Not very accurate with good reason. Ex: Average mileage 14 000 may vary and will affect the total cost

Question		Answers	Notes	Tot al
8	a	.1 identify the base of the triangle .2 add the squares of base and height .3 square root the sum of squares AG 5	.1 4-1 or 3 seen .1 ACCEPT triangle indicated on diagram .2 $3^2 + 4^2$ seen .3 $\sqrt{25}$, ACCEPT $3^2 + 4^2 = 5^2$	3
	b	correctly place 13 and 16	<table><tr><th>Stage (n)</th><th>Side (S)</th></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>7</td></tr><tr><td>4</td><td>10</td></tr><tr><td>5</td><td>13</td></tr><tr><td>6</td><td>16</td></tr></table>	1
	c	correctly describe one pattern for S in words with correct terminology	ACCEPT adding 3 OE difference 3 OE DO NOT ACCEPT The rule in words ex: subtract 2 from triple of stage OE $S = 3n - 2$ S is increasing odd even odd even arithmetic linear **Note:** DO NOT ACCEPT more than one pattern if any is incorrect Ex: Add 3 so they are all odd Add 3, they are multiples of 3 ACCEPT more than one pattern if all correct Ex: Add 3 so $3n - 2$ S is increasing, difference is 3	1

d	.1 the correct general rule .2 the correct simplified general rule with correct notation	.1 $3n-2$ or $S = 3 \times n - 2$ or $y = 3x - 2$ or $S = 1 + 3(n-1)$ .2 $S = 3n - 2$, ACCEPT $S_n = 3n - 2$ DO NOT ACCEPT description in words **SC** if NR in 8d and $(S =)3n - 2$ is seen in 8c award 1 mark		**2**
e	.1 correctly substitute $n \geq 5$ into their general rule .2 correctly calculate their value of S after substituting $n \geq 5$ .3 recognise that their correctly calculated value of S is the same as their predicted value	.1 Ex: $3 \times 5 - 2$ .2 Ex: 13 (for n=5) .3 same as value I predicted in table (and we find the candidate has 13 in the table for $n = 5$) **OR** same as when we continue the pattern and explains how 13 is obtained from pattern of adding 3 to 10 .3 ACCEPT seeing the 13 in the table in 8b and seeing their calculated S = 13 when $n = 5$		**3**

8	g	Mark	1	2	3	4	
		Predictions (P)	Correctly predict two terms for *P* ACCEPT whether in the table or in the response box	Correctly predict four terms for *P* ACCEPT whether in the table or in the response box			
		Description (D)	Attempt to describe a pattern in words for *P* OR a rule in words Ex: Constant/same difference Common difference Arithmetic sequence They are all even add 8 to 6 times n **OR** Attempt to describe pattern for T as general rule Ex for rule attempt: (P =) 6n or (P=)an+8 **OR** Correctly describe in words their pattern	Correctly describe one pattern in words for P Ex: increases by 6 adding 6 every time difference is always 6 arithmetic sequence with d=6 **OR** Correctly describe the pattern for *P* as a general rule $P = 6n + 8$ ACCEPT the rule is 6*n* +8 ACCEPT non-simplified rules Ex: 4 + (3(*n* + 1) – 2) + 5 + 3*n* – 2	Correctly describe one pattern in words for P **AND** Correctly describe the pattern for *P* as a general rule $P = 6n + 8$ ACCEPT the rule is 6*n* +8 ACCEPT non-simplified rules Ex: 4 + (3(*n* + 1) – 2) + 5 + 3*n* – 2		20
		Testing (T)	Attempt to test their general rule for *P* using $n \leq 4$ Ex: correctly substitute in their general rule value of *n* ≤ 4 **OR**	Correctly test their general rule for *P* using $n \leq 4$ Ex: Correctly calculate their value for P in their general rule using $n \leq 4$ **AND** Recognise that their correctly calculated value for *P* is the same as the given value.			

			Correctly test their described pattern or their rule (e.g. recursive rule)	ACCEPT seeing their correctly calculated value for P and the given value in the table being equal		
		Verifying (V)	Attempt to verify their general rule for P using $n \geq 5$ Ex: correctly substitute in their general rule value of $n \geq 5$ **OR** Correctly verify their described pattern or their rule (e.g. recursive rule)	Correctly calculate their value for P in their general rule using $n \geq 5$	Correctly calculate their value for P in their general rule using $n \geq 5$ **AND** Recognise that their correctly calculated value for P is the same as their predicted value obtained by continuing the pattern ACCEPT seeing their correctly calculated value for P and their predicted value in the table being equal	
		Justify/proof (J)	Attempt to justify their described pattern or their general rule Ex: trying at least two more values and arguing as justification that they are the same or rule works or It is linear or arithmetic seen as justification	Justify their general rule arithmetically Ex: It is an arithmetic sequence with difference 6 and first term 14 or 14+(n-1)6 **OR** Attempt to justify their general rule geometrically Ex: Since the side increases by 3 and we added another side that increases by 3 so the increase is 6 or the lengths of the rectangle increase by 3 each or	Good attempt to justify the general rule for P geometrically by using incorrect four lengths in terms of n $4 + 3n - 2 + 5 + 3n - 2$ or $3 + 4 + 5 + 2(3n)$ OE	Correctly justify the general rule for P geometrically by using correct lengths in terms of n $4 + (3(n + 1) - 2) + 5 + 3n - 2$ or $3 + 4 + 5 + 2(3n - 2)$ OE J4 automatically gains T2 and V3

			weak attempt to get perimeter using three lengths in terms of n	
Notation and terminology (N)	Correct notation of their rule **OR** Correct terminology describing at least one pattern	Correct notation of the general rule for P **OR** The notation of the general rule includes errors **AND** Correct terminology describing at least one pattern	Correct notation of the general rule for P **AND** Correct terminology describing at least one pattern	
	DO NOT ACCEPT if they don't have any rules and they don't describe any patterns	DO NOT ACCEPT if they don't have a general rule	The general rule: $P = 6n + 8$ For notation of general rule, DO NOT ACCEPT $P = 6 \times n + 8$ The rule for P is $6n + 8$ Correct non simplified rules, Ex: 4 + (3(n + 1) − 2) + 5 + 3n − 2	
Communication (L)	Very weak communication Two or three lines of communication **OR** Only calculations or algebraic steps	Weak communication More than three lines of communication but lack coherence	Good communication More than three lines of coherent communication Can be awarded only if J2 is achieved	

Predictions

Stage (*n*)	Side (*S*)	Perimeter (*P*)
1	1	14
2	4	20
3	7	26
4	10	32
5	13	38
6	16	44
7	19	50
8	22	56